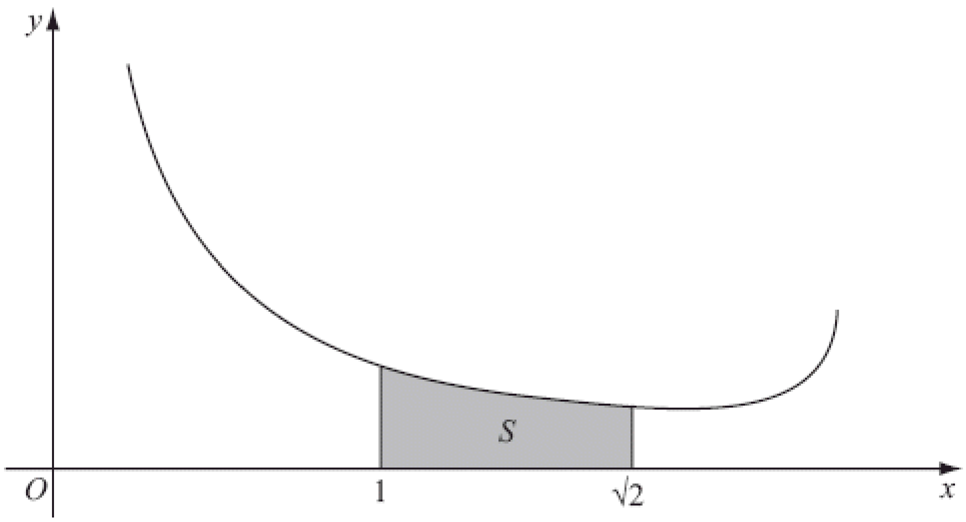
1. (a) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,\mathrm{d}x$$

(7)



The diagram above shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{4}}}$,

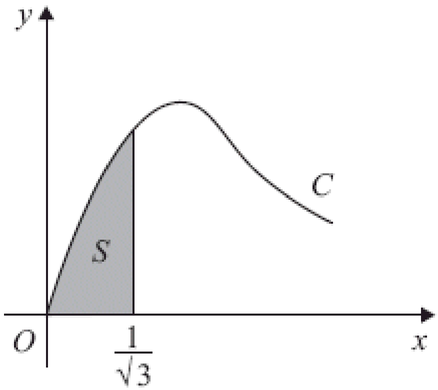
$0 < x < 2$.

The shaded region S, shown in the diagram above, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

(3)
(Total 10 marks)

2. (a) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \,\mathrm{d}\theta$.



(2)

The diagram above shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = 2\sin 2\theta, \qquad 0 \le \theta < \frac{\pi}{2}$$

The finite shaded region S shown in the diagram is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$$

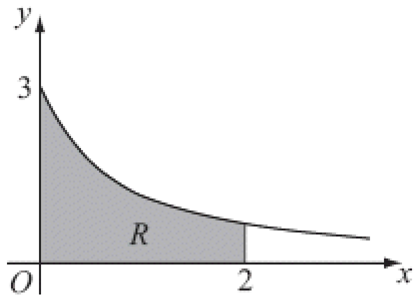
where k is a constant.

(5)

(c) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)
(Total 10 marks)

3.



The diagram above shows part of the curve $y = \frac{3}{\sqrt{(1+4x)}}$. The region R is bounded by the curve, the x-axis, and the lines $x = 0$ and $x = 2$, as shown shaded in the diagram above.

(a) Use integration to find the area of R.

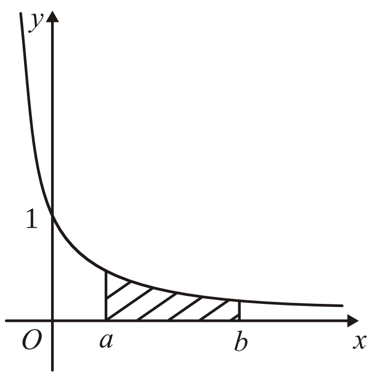
(4)

The region R is rotated 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid formed.

(5)
(Total 9 marks)

4.



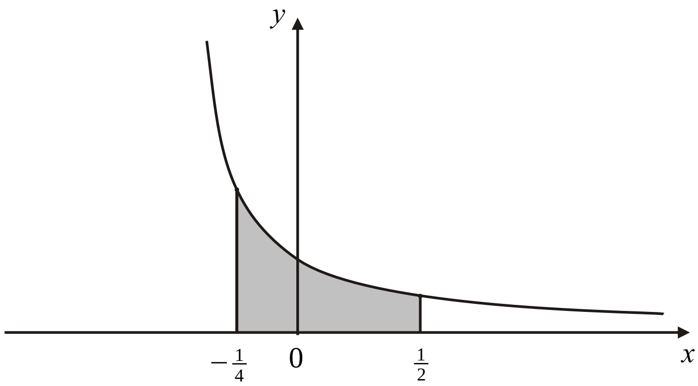
The curve shown in the diagram above has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in the diagram. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(Total 5 marks)

5.

Figure 1



The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\dfrac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\dfrac{1}{4}$, $x = \dfrac{1}{2}$, the x-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

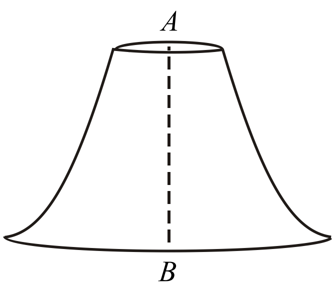
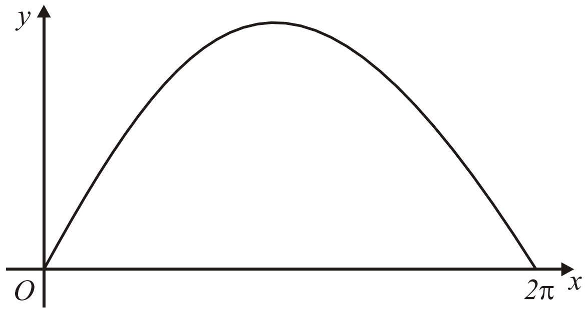


Figure 2 shows a paperweight with axis of symmetry AB where AB = 3 cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight.
The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)
(Total 7 marks)

6.



The curve with equation, $y = 3\sin\frac{x}{2}, 0 \leq x \leq 2\pi$, is shown in the figure above. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

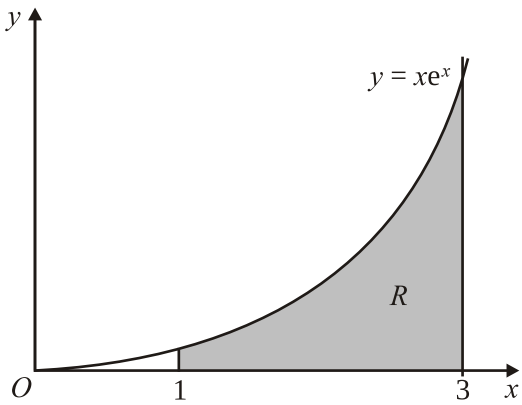
(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)
(Total 9 marks)

7.



The figure above shows the finite shaded region, R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(Total 8 marks)

8.

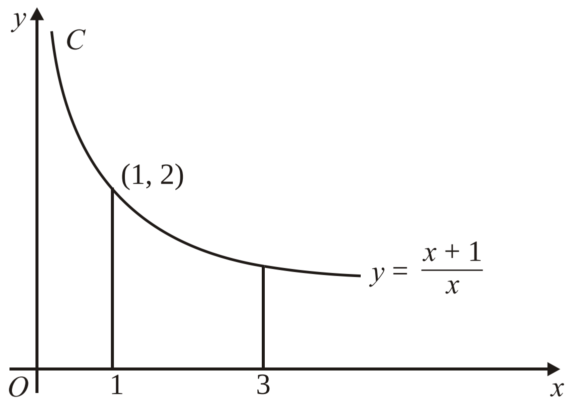


Figure 1

Figure 1 shows part of the curve C with equation $y = \frac{x+1}{x}$, $x > 0$.

The finite region enclosed by C, the lines $x = 1$, $x = 3$ and the x-axis is rotated through 360° about the x-axis to generate a solid S.

(a) Using integration, find the exact volume of S.

(7)

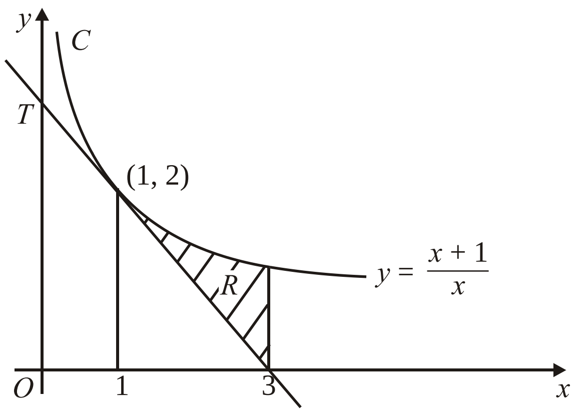


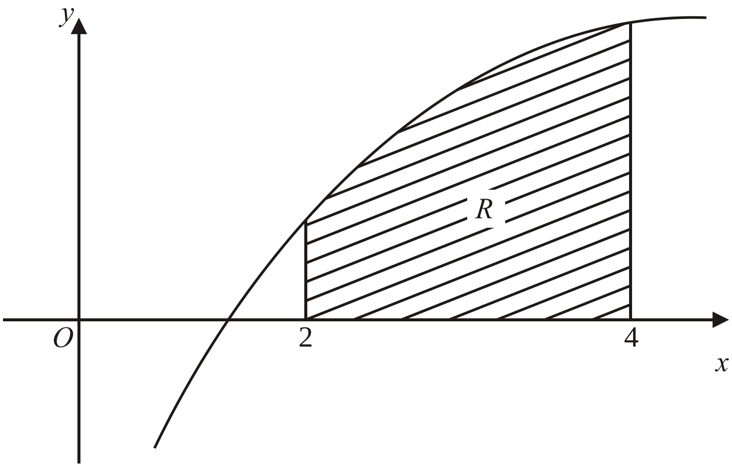
Figure 2

The tangent T to C at the point (1, 2) meets the x-axis at the point (3, 0). The shaded region R is bounded by C, the line $x = 3$ and T, as shown in Figure 2.

(b) Using your answer to part (a), find the exact volume generated by R when it is rotated through 360° about the x-axis.

(3)
(Total 10 marks)

9.



The diagram above shows part of the curve with equation

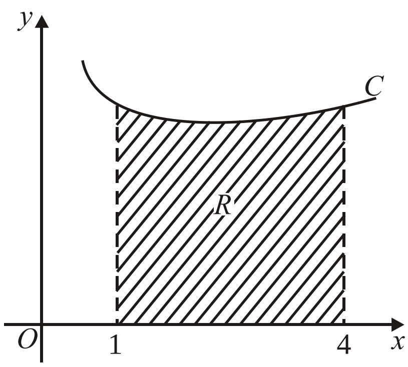
$$y = 4x - \frac{6}{x}, \quad x > 0.$$

The shaded region R is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 4$. This region is rotated through 2π radians about the x-axis.

Find the exact value of the volume of the solid generated.

(Total 8 marks)

10.



The diagram above shows parts of the curve C with equation

$$y = \frac{x+2}{\sqrt{x}}.$$

The shaded region R is bounded by C, the x-axis and the lines $x = 1$ and $x = 4$.

This region is rotated through 360° about the x-axis to form a solid S.

(a) Find, by integration, the exact volume of S.

(7)

The solid S is used to model a wooden support with a circular base and a circular top.

(b) Show that the base and the top have the same radius.

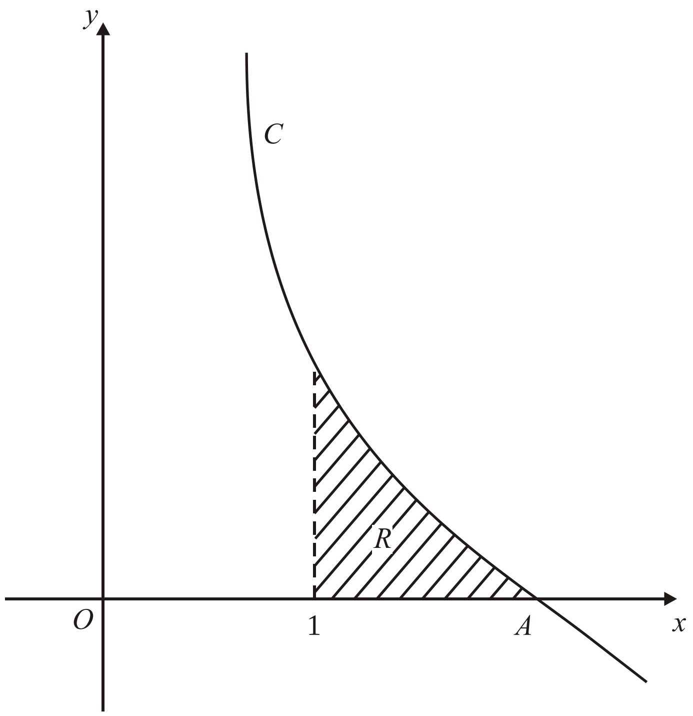
(1)

Given that the actual radius of the base is 6 cm,

(c) show that the volume of the wooden support is approximately 630 cm^3.

(2)
(Total 10 marks)

11.



The diagram above shows the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{8}{x} - x^2,\ \ x > 0.$$

Given that C crosses the x-axis at the point A,

(a) find the coordinates of A.

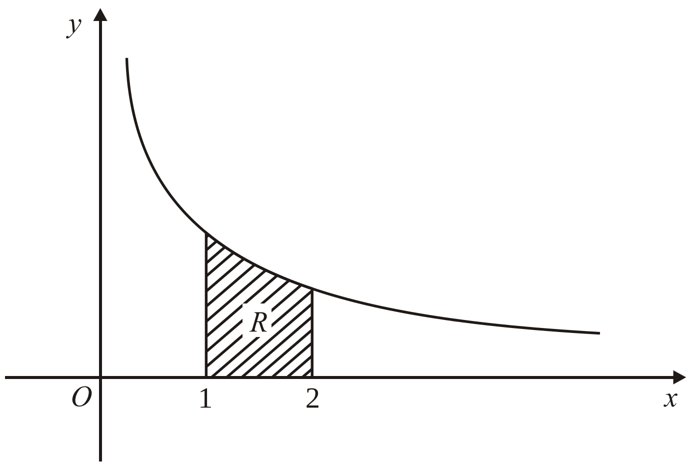
(3)

The finite region R, bounded by C, the x-axis and the line $x = 1$, is rotated through 2π radians about the x-axis.

(b) Use integration to find, in terms of π the volume of the solid generated.

(7)
(Total 10 marks)

12.



The diagram above shows part of the curve with equation $y = 1 + \frac{c}{x}$, where c is a positive constant.

The point P with x-coordinate p lies on the curve. Given that the gradient of the curve at P is -4,

(a) show that $c = 4p^2$.

(2)

Given also that the y-coordinate of P is 5,

(b) prove that $c = 4$.

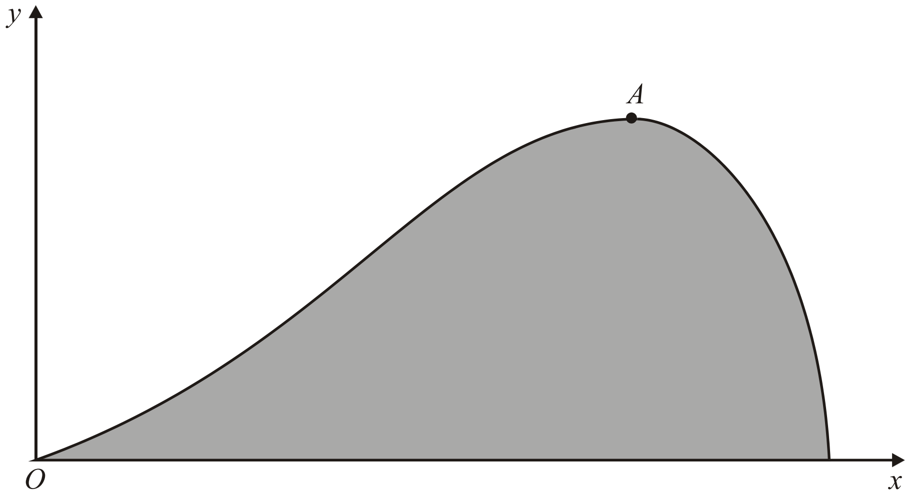
(2)

The region R is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$, as shown in the diagram above. The region R is rotated through 360° about the x-axis.

(c) Show that the volume of the solid generated can be written in the form $\pi(k + q \ln 2)$, where k and q are constants to be found.

(7)
(Total 11 marks)

13.



The diagram above shows a graph of $y = x\sqrt{\sin x}$, $0 < x < \pi$. The maximum point on the curve is A.

(a) Show that the x-coordinate of the point A satisfies the equation $2 \tan x + x = 0$.

(4)

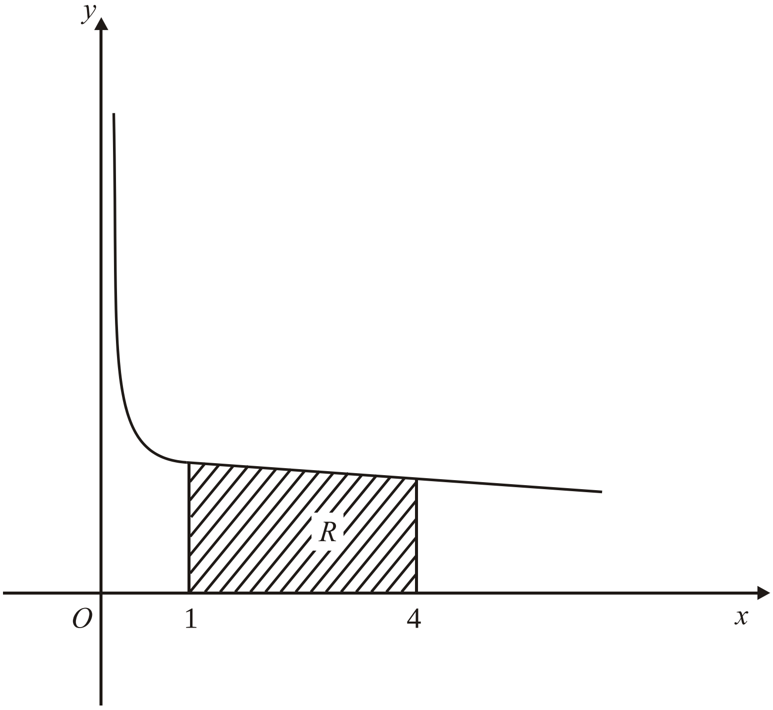
The finite region enclosed by the curve and the x-axis is shaded as shown in the diagram above.

A solid body S is generated by rotating this region through 2π radians about the x-axis.

(b) Find the exact value of the volume of S.

(7)
(Total 11 marks)

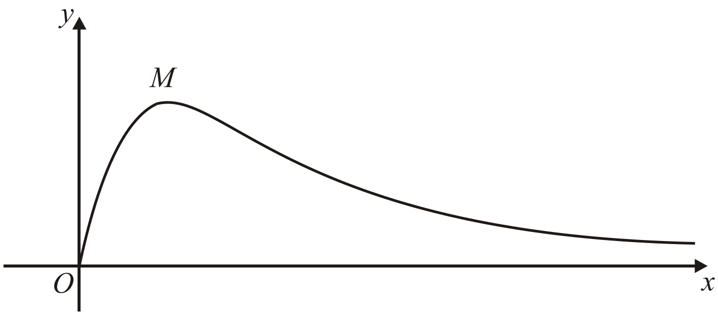
14.



The diagram above shows part of the curve with equation $y = 1 + \dfrac{1}{2\sqrt{x}}$. The shaded region R, bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$, is rotated through 360° about the x-axis. Using integration, show that the volume of the solid generated is $\pi\left(5 + \frac{1}{2}\ln 2\right)$.

(Total 8 marks)

15.



The diagram above shows the curve with equation $y = x^{\frac{1}{2}}e^{-2x}$.

(a) Find the x-coordinate of M, the maximum point of the curve.

(5)

The finite region enclosed by the curve, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

(b) Find, in terms of π and e, the volume of the solid generated.

(7)
(Total 12 marks)

Figure 2 shows a paperweight with axis of symmetry AB where AB = 3 cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight.
The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)
(Total 7 marks)

16. Find the volume generated when the region bounded by the curve with equation $y = 2 + \frac{1}{x}$, the x-axis and the lines $x = \frac{1}{2}$ and $x = 4$ is rotated through 360° about the x-axis.

Give your answer in the form $\pi(a + b \ln 2)$, where a and b are rational constants.

……………………………..……..……………………………………………………………..

(Total 7 marks)

1. (a) $\dfrac{dx}{du} = -2\sin u$ B1

$\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,du$ M1

$\displaystyle = \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,du$ Use of $1 - \cos^2 u = \sin^2 u$ M1

$\displaystyle = -\frac{1}{4}\int \frac{1}{\cos^2 u}\,du$ $\displaystyle \pm k\int \frac{1}{\cos^2 u}\,du$ M1

$\displaystyle = -\frac{1}{4}\tan u(+C)$ $\pm k \tan u$ M1

$x = \sqrt{2} \Rightarrow \sqrt{2} = 2\cos u \Rightarrow u = \dfrac{\pi}{4}$

$x = 1 \Rightarrow 1 = 2\cos u \Rightarrow u = \dfrac{\pi}{3}$ M1

$\displaystyle \left[-\frac{1}{4}\tan u\right]_{\frac{\pi}{4}}^{\frac{\pi}{4}} = -\frac{1}{4}\left(\tan\frac{\pi}{4} - \tan\frac{\pi}{3}\right)$

$\displaystyle = -\frac{1}{4}\left(1-\sqrt{3}\right)\left(= \frac{\sqrt{3}-1}{4}\right)$ A1 7

(b) $\displaystyle V = \pi\int_1^{\sqrt{2}} \left(\frac{4}{x(4-x^2)^{\frac{1}{4}}}\right)^2 dx$ M1

$\displaystyle = 16\pi\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\,dx$ $16\pi \times$ integral in (a) M1

$\displaystyle = 16\pi\left(\frac{\sqrt{3}-1}{4}\right)$ $16\pi \times$ their answer to part (a) A1ft 3

[10]

2. (a) $\displaystyle\int \sin^2\theta\,d\theta = \frac{1}{2}\int (1-\cos 2\theta)\,d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\,(+C)$ M1 A1 2

(b) $x = \tan\theta \Rightarrow \dfrac{dx}{d\theta} = \sec^2\theta$

$\displaystyle \pi\int y^2\,dx = \pi\int y^2\frac{dx}{d\theta}\,d\theta = \pi\int (2\sin 2\theta)^2\sec^2\theta\,d\theta$ M1 A1

$$= \pi \int \frac{(2 \times 2 \sin\theta \cos\theta)^2}{\cos^2\theta} \mathrm{d}\theta$$ M1

$$16\pi \int \sin^2\theta \, \mathrm{d}\theta$$ $k = 16\pi$ A1

$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0,\ x = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \dfrac{\pi}{6}$ B1 5

$$\left(V = 16\pi \int_0^{\frac{\pi}{6}} \sin^2\theta \, \mathrm{d}\theta \right)$$

(c) $V = 16\pi \left[\dfrac{1}{2}\theta - \dfrac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{6}}$ M1

$= 16\pi \left[\left(\dfrac{\pi}{12} - \dfrac{1}{4}\sin\dfrac{\pi}{3} \right) - (0 - 0) \right]$ Use of correct limits *M1

$= 16\pi \left(\dfrac{\pi}{12} - \dfrac{\sqrt{3}}{8} \right) = \dfrac{4}{3}\pi^2 - 2\pi\sqrt{3}$ $p = \dfrac{4}{3}, q = -2$ A1 3

[10]

3. (a) Area(R) = $\displaystyle\int_0^2 \frac{3}{\sqrt{(1+4x)}} \mathrm{d}x = \int_0^2 3(1+4x)^{-\frac{1}{2}} \mathrm{d}x$

Integrating $3(1 + 4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$. M1

$= \left[\dfrac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2}.4} \right]_0^2$ <u>Correct integration</u>. Ignore limits. A1

$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}} \right]_0^2$

$= \left(\frac{3}{2}\sqrt{9}\right) - \left(\frac{3}{2}(1)\right)$ Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round. M1

$= \frac{9}{2} - \frac{3}{2} = \underline{3}(\text{units})^2$ <u>3</u> <u>A1</u> 4

(Answer of 3 with no working scores M0A0M0A0.)

(b) Volume = $\underline{\pi \displaystyle\int_0^2 \left(\frac{3}{\sqrt{(1+4x)}} \right)^2 \mathrm{d}x}$ Use of $V = \underline{\pi \int y^2}\, \mathrm{d}x$ ..

Can be implied. Ignore limits and dx. B1

$$= (\pi)\int_0^2 \frac{9}{1+4x}\,dx$$

$$= (\pi)\left[\tfrac{9}{4}\ln|1+4x|\right]_0^2$$ $\pm k\ln|1+4x|$ M1

$\tfrac{9}{4}\ln|1+4x|$ A1

$$= (\pi)\left[(\tfrac{9}{4}\ln 9) - (\tfrac{9}{4}\ln 1)\right]$$ Substitutes limits of 2 and 0 and subtracts the correct way round. dM1

Note that ln1 can be implied as equal to 0.

So Volume = $\underline{\tfrac{9}{4}\pi\ln 9}$ $\underline{\tfrac{9}{4}\pi\ln 9}$ or $\underline{\tfrac{9}{2}\pi\ln 3}$ or $\underline{\tfrac{18}{4}\pi\ln 3}$ A1 oe isw 5

Note the answer must be a one term exact value. Note that $\tfrac{9}{4}\pi\ln 9 + c$ (oe.) would be awarded the final A0.

Note, also you can ignore subsequent working here.

[9]

4. Volume = $\underline{\pi\int_a^b \left(\frac{1}{2x+1}\right)^2 dx} = \pi\int_a^b \frac{1}{(2x+1)^2}\,dx$

$$= \pi\int_a^b (2x+1)^{-2}\,dx$$

$$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$$

$$= (\pi)\left[\underline{-\frac{1}{2}(2x+1)^{-1}}\right]_a^b$$

$$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$$

Use of $V = \underline{\pi\int y^2\,dx}$.

Can be implied. Ignore limits. B1

Integrating to give $\underline{\pm p(2x+1)^{-1}}$ M1

$\underline{-\frac{1}{2}(2x+1)^{-1}}$ A1

Substitutes limits of b and a and subtracts the correct way round. dM1

$\dfrac{\pi(b-a)}{(2a+1)(2b+1)}$ (*) A1 aef 5

(*) Allow other equivalent forms such as

$\dfrac{\pi b-\pi a}{(2a+1)(2b+1)}$ or $\dfrac{-\pi(a-b)}{(2a+1)(2b+1)}$ or $\dfrac{\pi(b-a)}{4ab+2a+2b+1}$ or $\dfrac{\pi b-\pi a}{4ab+2a+2b+1}$.

Note that n is not required for the middle three marks of this question.

Aliter
Way 2

Volume = $\pi\int_a^b\left(\dfrac{1}{2x+1}\right)^2\mathrm{d}x = \pi\int_a^b\dfrac{1}{(2x+1)^2}\mathrm{d}x$

$= \pi\int_a^b(2x+1)^{-2}\mathrm{d}x$

Use of $V = \pi\int y^2\,\mathrm{d}x$.

Can be implied. Ignore limits. B1

Applying substitution $u = 2x + 1 \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a + 1$ and $b \to 2b + 1$, gives

$$= (\pi)\int_{2a+1}^{2b+1}\frac{u^{-2}}{2}\mathrm{d}u$$

$$= (\pi)\left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$$

$$= (\pi)\left[-\frac{1}{2}u^{-1}\right]_{2a+1}^{2b+1}$$

$$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right)-\left(\frac{-1}{2(2a+1)}\right)\right]$$

$$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$$

$$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$$

Integrating to give $\underline{\pm pu^{-1}}$ M1

$\underline{-\frac{1}{2}u^{-1}}$ A1

Substitutes limits of $2b + 1$ and $2a + 1$ and subtracts the correct way round. dM1

$\underline{\dfrac{\pi(b-a)}{(2a+1)(2b+1)}}$ (*) A1 aef 5

(*) Allow other equivalent forms such as

$\dfrac{\pi b-\pi a}{(2a+1)(2b+1)}$ or $\dfrac{-\pi(a-b)}{(2a+1)(2b+1)}$ or $\dfrac{\pi(b-a)}{4ab+2a+2b+1}$ or $\dfrac{\pi b-\pi a}{4ab+2a+2b+1}$.

Note that π is not required for the middle three marks of this question.

[5]

5. (a) Volume = $\underline{\pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\left(\dfrac{1}{3(1+2x)}\right)^2 dx} = \dfrac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}}\dfrac{1}{(1+2x)^2}dx$

$$= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}}(1+2x)^{-2}dx$$

$$= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{2}(1+2x)^{-1}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right)-\left(\frac{-1}{2\left(\frac{1}{2}\right)}\right)\right]$$

$$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{4}-(-1)\right]$$

$$= \frac{\pi}{12}$$

Use of $\underline{V = \pi\int y^2 dx}$

Can be implied. Ignore limits. B1

Moving their power to the top.
(Do not allow power of –1.)

Can be implied. Ignore limits and $\dfrac{\pi}{9}$ M1

Integrating to give $\underline{\pm p(1+2x)^{-1}}$	M1	
$\underline{-\frac{1}{2}(1+2x)^{-1}}$	A1	
Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1aef	5

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks.

Aliter
Way 2

$$\text{Volume} = \underline{\pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\left(\frac{1}{3(1+2x)}\right)^2 \mathrm{d}x} = \pi\int_{-\frac{1}{4}}^{\frac{1}{2}}\frac{1}{(3+6x)^2}\mathrm{d}x$$

$$= (\pi)\int_{-\frac{1}{4}}^{\frac{1}{2}}(3+6x)^{-2}\mathrm{d}x$$

$$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= (\pi)\left[\underline{-\frac{1}{6}(3+6x)^{-1}}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$$

$$= (\pi)\left[\left(\frac{-1}{6(6)}\right)-\left(\frac{-1}{6\left(\frac{3}{2}\right)}\right)\right]$$

$$= (\pi)\left[-\frac{1}{36}-(-\frac{1}{9})\right]$$

$$= \frac{\pi}{12}$$

Use of $V = \underline{\pi\int y^2\,\mathrm{d}x}$ Can be implied. Ignore limits.	B1	
Moving their power to the top. **(Do not allow power of –1.)** Can be implied. Ignore limits and π	M1	
Integrating to give $\underline{\pm p(3+6x)^{-1}}$	M1	
$\underline{-\frac{1}{6}(3+6x)^{-1}}$	A1	
Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1aef	5

Note: π (or implied) is not needed for the middle three marks.

(b) From Fig.1, AB = $\frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units

As $\frac{3}{4}$ units ≡ 3cm

then scale factor k = $\frac{3}{\left(\frac{3}{4}\right)}$ = 4.

Hence Volume of paperweight = $(4)^3\left(\frac{\pi}{12}\right)$

V = $\underline{\frac{16\pi}{3}}$ cm^3 =16.75516... cm^3

$(4)^3$ × (their answer to part (a))	M1	
$\underline{\frac{16\pi}{3}}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef	A1	2

[7]

6. (a) Area Shaded = $\int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\mathrm{d}x$

$= \left[\frac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{-2\pi}$

Integrating 3sin($\frac{x}{2}$) to give k cos ($\frac{x}{2}$) with k ≠ 1. Ignore limits. M1

$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$

$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$ A1 oe

$= \left[-6(-1)\right] - \left[-6(1)\right] = 6 + 6 = \underline{12}$ A1 cao 3

(Answer of 12 with no working scores M0A0A0.)

(b) Volume = $\underline{\pi\int_0^{2\pi}\left(3\sin\left(\frac{x}{2}\right)\right)^2 \quad \mathrm{d}x} = 9\pi\int_0^{2\pi}\sin^2\left(\frac{x}{2}\right)\mathrm{d}x$

Use of $V = \pi\int y^2\,\mathrm{d}x$.
Can be implied. Ignore limits. M1

[NB: cos2x = ±1 ±2sin^2x gives sin^2x $x = \frac{1-\cos 2x}{2}$]

[NB: $\cos x = \pm 1 \pm 2\sin^2(\frac{x}{2})$ gives $\sin^2(\frac{x}{2}) = \frac{1-\cos x}{2}$] M1

Consideration of the Half Angle Formula for $\sin^2(\frac{x}{2})$ or the Double Angle Formula for $\sin^2 x$

$\therefore$ Volume $= 9(\pi)\int_0^{2\pi}\left(\frac{1-\cos x}{2}\right)dx$ A1

Correct expression for Volume
Ignore limits and π.

$= \frac{9(\pi)}{2}\int_0^{2\pi}(\underline{1-\cos x})\,dx$

$= \frac{9(\pi)}{2}\left[x - \sin x\right]_0^{2\pi}$

Integrating to give $\pm ax \pm b\sin x$; depM1;
Correct integration
$k - k\cos x \to kx - k\sin x$ A1

$= \frac{9\pi}{2}\left[(2\pi - 0) - (0 - 0)\right]$

$= \frac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264.}$ A1 cso 3

Use of limits to give either $9\pi^2$ or awrt 88.8
Solution must be completely correct. No flukes allowed.

[6]

7. Attempts $V = \pi\int x^2e^{2x}\,dx$ M1

$= \pi\left[\frac{x^2e^{2x}}{2} - \int xe^{2x}\,dx\right]$ (M1 needs parts in the correct direction) M1 A1

$= \pi\left[\frac{x^2e^{2x}}{2} - \int xe^{2x}\,dx\right]$ (M1 needs second application of parts) M1 A1ft

M1A1ft refers to candidates $\int xe^{2x}\,dx$, but dependent on prev. M1

$= \pi\left[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \int\frac{e^{2x}}{4}\right)\right]$ A1 cao

Substitutes limits 3 and 1 and subtracts to give... dM1
[dep. on second and third Ms]

$= \pi\left[\frac{13}{4}e^6 - \frac{1}{4}e^2\right]$ or any correct exact equivalent. A1 8

[Omission of π loses first and last marks only]

[8]

8. (a) $\left(\frac{x+1}{x}\right)^2 = 1 + \frac{2}{x} + \frac{1}{x^2}$ anywhere B1

$V = \pi\int\left(\frac{x+1}{x}\right)^2 \mathrm{d}x$ M1

$\int\left(\frac{x+1}{x}\right)^2 \mathrm{d}x = x - \frac{1}{x}, +2\ln x$ M1A1,A1

M1 attempt to ∫

Using limits correctly in their integral:

$(\pi) = \{[x + 2\ln x - \frac{1}{x}]^3 - [x + 2\ln x - \frac{1}{x}]_1\}$ M1

$V = \pi[2^2/_3 + 2\ln 3]$ A1 7

Must be exact

(b) Volume of cone (or vol. generated by line) = $\frac{1}{3}\pi \times 2^2 \times 2$ B1

$V_R = V_S -$ volume of cone $= V_S - {}^1/_3\pi \times 2^2 \times 2$ M1

$= 2\pi\ln 3$ or $\pi\ln 9$ A1 3

[10]

9. Use of $V = \pi\int y^2 \mathrm{d}x$ M1

$y^2 = 16x^2 + \frac{36}{x^2}$; -48

Integrating to obtain $(\pi)\left[\frac{16x^3}{3} - \frac{36}{x}; -48x\right]$ ft constants only M1 A1ft; A1ft

$(\pi)\left[\frac{16x^3}{3} - \frac{36}{x} - 48x\right]_2^4 = (\pi)\left[140\frac{1}{3} - (-71\frac{1}{3})\right]$ correct use of limits M1

$V = 211\frac{2}{3}\pi$ (units^3) A1 8

[8]

10. (a) $y^2 = \left(\frac{x+2}{\sqrt{x}}\right)^2 = \frac{x^2 + 4x + 4}{x} = x + 4 + \frac{4}{x}$ M1A1

$\pi\int y^2 \mathrm{d}x$ [dependent on attempt at squaring y] B1

$\int y^2 \mathrm{d}x = \int (\frac{x^2 + 4x + 4}{x})\mathrm{d}x$; $= \frac{x^2}{2} + 4x + 4\ln x$ M1;A1 ft

[A1√ must have ln x term]

Correct use of limits: $[\ ]_1^4 = [\ ]^4 - [\ ]_1$ M1

[M dependent on prev. M1]

Volume = $\left(\frac{39}{2} + 4\ln 4\right)\pi$ or equivalent exact A1 7

(b) Showing that $y = 3$ at $x = 1$ and $x = 4$ B1 1

(c) Volume = 2^3 × answer to (a) ; = 629.5 cm^3 $\approx$ 630 cm^3 (*) M1;A1 2

[allow 629 – 630]

[10]

11. (a) $\frac{8}{x} - x^2 = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$ M1 A1 2

(b) $\left(\frac{8}{x} - x^2\right)^2 = x^4 - 16x + \frac{64}{x^2}$ M1

$\int (x^4 - 16x + 64x^{-2})\mathrm{d}x = \frac{x^5}{5} - 8x^2 - \frac{64}{x}$ M1 A1

$\left[\frac{x^5}{5} - 8x^2 - \frac{64}{x}\right]_1^2 = \left(\frac{32}{5} - 32 - 32\right) - \left(\frac{1}{5} - 8 - 64\right)$ M1 A1 ft

Volume is $\frac{71}{5}\pi$ (units^3) A1 7

[9]

12. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{c}{x^2}$ M1

Attempt $\frac{\mathrm{d}y}{\mathrm{d}x}$

When $x = p \Rightarrow -4 = -\frac{c}{p^2}$ $\therefore$ <u>$c = 4p^2$</u> (*) A1 cso 2

(b) $5 = 1 + \dfrac{c}{p}$ and solve with $c = 4p^2$ M1

$5 = 1 + 4p \Rightarrow p = 1 \therefore \underline{c = 4}$ (*) A1 cso 2

(c) $y^2 = 1 + \dfrac{8}{x} + \dfrac{16}{x^2}$ M1

$y^2 = ;\ \geq 2$ terms correct

$$\int y^2 \mathrm{d}x = \left[x + 8\ln x - \frac{16}{x}\right]$$

some correct $\int$ M1

all correct A1

$$\int_1^2 y^2 \mathrm{d}x = (2 + 8\ln 2 - \frac{16}{2}) - (1 + 8\ln 1 - 16)$$ M1

Use of correct limits

$= 9 + 8\ln 2$

$V = \pi \int_1^2 y^2 \mathrm{d}x$; $\therefore$ $\underline{V = \pi(9 + 8\ln 2)}$

$V = \pi\int y^2 dx$ B1

$k = 9$; A1

$q = 8$ A1 7

[11]

13. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\sin x} + \dfrac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x$ M1, A1

At A $\sqrt{\sin x} + \dfrac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x = 0$ dM1

$\therefore \sin x + \dfrac{x}{2}\cos x = 0$ (essential to see intermediate line before given answer)

$\therefore 2\tan x + x = 0$ (*) A1 4

(b) $V = \pi\int y^2 \mathrm{d}x = \pi\int x^2 \sin x \mathrm{d}x$ M1

$= \pi\left[-x^2 \cos x + \int 2x\cos x \mathrm{d}x\right]_0^\pi$ M1 A1

$= \pi\left[-x^2 \cos x + 2x\sin x - \int 2\sin x \mathrm{d}x\right]_0^\pi$ M1

$= \pi\left[-x^2 \cos x + 2x\sin x + 2\cos x\right]_0^\pi$ A1

$= \pi[\pi^2 - 2 - 2]$ M1

$= \pi[\pi^2 - 4]$ A1 7

[11]

14. Volume = $\pi \int_1^4 \left(1 + \frac{1}{2\sqrt{x}}\right)^2 dx$ M1

$\int \left(1 + \frac{1}{2\sqrt{x}}\right)^2 dx = \int \left(1 + \frac{1}{\sqrt{x}} + \frac{1}{4x}\right) dx$ B1

$= \left[x + 2\sqrt{x} + \frac{1}{4}\ln x\right]$ M1 A1 A1 ft

Using limits correctly M1

Volume = $\pi\left[\left(8 + \frac{1}{4}\ln 4\right) + 3\right]$ A1

$= \pi\left[5 + \frac{1}{2}\ln 2\right]$ A1 8

[8]

15. (a) $\frac{dy}{dx} = 2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$ M1 A1 A1

Putting $\frac{dy}{dx}$ = 0 and attempting to solve dM1

$x = \frac{1}{4}$ A1 5

(b) Volume = $\pi \int_0^1 (\sqrt{x}e^{-2x})^2\, dx = \pi \int_0^1 xe^{-4x} dx$ M1 A1

$\int xe^{-4x} dx = -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx$ M1 A1

$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$ A1 ft

Volume = $\pi\left[-\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4}\right] - \left[-\frac{1}{16}\right] = \frac{\pi}{16}[1 - 5e^{-4}]$ M1 A1 7

[12]

16. Volume = $\pi \int_{\frac{1}{2}}^{4} \left(2 + \frac{1}{x}\right)^2 dx$, $= \pi \int_{\frac{1}{2}}^{4} \left(4 + \frac{4}{x} + \frac{1}{x^2}\right) dx$ M1, M1

$= \pi\left[4x + 4\ln x - x^{-1}\right]_{\frac{1}{2}}^{4}$ A1 A1

$= \pi\left[16 + 4\ln 4 - \frac{1}{4} - 2 - 4\ln\frac{1}{2} + 2\right]$ M1 A1

$= \pi[15.75 + 12 \ln 2]$ A1

[7]

1. Answers to part (a) were mixed, although most candidates gained some method marks. A surprisingly large number of candidates failed to deal with $\sqrt{4-4\cos^2 u}$ correctly and many did not recognise that $\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx = \tan x(+C)$ in this context. Nearly all converted the limits correctly. Answers to part (b) were also mixed. Some could not get beyond stating the formula for the volume of revolution while others gained the first mark, by substituting the equation given in part (b) into this formula, but could not see the connection with part (a). Candidates could recover here and gain full follow through marks in part (b) after an incorrect attempt at part (a).

2. The responses to this question were very variable and many lost marks through errors in manipulation or notation, possibly through mental tiredness. For examples, many made errors in manipulation and could not proceed correctly from the printed $\cos 2\theta = 1 - 2\sin^2\theta$ to $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and the answer $\frac{x}{2} - \frac{1}{4}\sin 2\theta$ was often seen, instead of $\frac{\theta}{2} - \frac{1}{4}\sin 2\theta$. In part (b), many never found $\frac{dx}{d\theta}$ or realised that the appropriate form for the volume was $\pi\int y^2 \frac{dx}{d\theta}\,d\theta$.

However the majority did find a correct integral in terms of θ although some were unable to use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify their integral. The incorrect value $k = 8\pi$ was very common, resulting from a failure to square the factor 2 in $\sin 2\theta = 2\sin\theta\cos\theta$. Candidates were expected to demonstrate the correct change of limits. Minimally a reference to the result $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, or an equivalent, was required. Those who had complete solutions usually gained the two method marks in part (c) but earlier errors often led to incorrect answers.

3. Q2 was generally well answered with many successful attempts seen in both parts. There were few very poor or non-attempts at this question.

In part (a), a significant minority of candidates tried to integrate $3(1+4x)^{\frac{1}{2}}$. Many candidates, however, correctly realised that they needed to integrate $3(1+4x)^{-\frac{1}{2}}$. The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $k(1+4x)^{\frac{1}{2}}$. Few candidates applied incorrect limits to their integrated expression. A noticeable number of candidates, however, incorrectly assumed a subtraction of zero when substituting for $x = 0$ and so lost the final two marks for this part. A minority of candidates attempted to integrate the expression in part (a) by using a substitution. Of these candidates, most were successful.

In part (b), the vast majority of candidates attempted to apply the formula $\pi\int y^2\,dx$, but a few of them were not successful in simplifying y^2. The majority of candidates were able to integrate $\frac{9}{1+4x}$ to give $\frac{9}{4}\ln|1+4x|$. The most common error at this stage was for candidates to omit dividing by 4. Again, more candidates were successful in this part in substituting the limits correctly to arrive at the exact answer of $\frac{9}{4}\pi\ln 9$. Few candidates gave a decimal answer with no exact term seen and lost the final mark.

4. Most candidates used the correct volume formula to obtain an expression in terms of x for integration. At this stage errors included candidates using either incorrect formulae of $\pi\int y\mathrm{d}x, 2\pi\int y^2\mathrm{d}x$ or $\int y^2\mathrm{d}x$. Many candidates realised that they needed to integrate an expression of the form $(2x + 1)^{-2}$ (or equivalent). The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $p(1 + 2x)^{-1}$. A few candidates, however, integrated to give an expression in terms of natural logarithms. A significant minority of candidates substituted the limits of $b$and a into their integrand the wrong way round. Only a minority of candidates were able to combine together their rational fractions to give an answer as a single simplified fraction as required by the question.

5. In part (a), most candidates used the correct volume formula to obtain an expression in terms of x for integration. At this stage errors included candidates using either incorrect formulae of $\pi\int y\mathrm{d}x$ or $\int y^2\mathrm{d}x$. Many candidates realised that they needed to integrate an expression of the form $k(1 + 2x)^{-2}$ (or equivalent). The majority of these candidates were able to complete the integration correctly or at least achieve an integrated expression of the form $p(1 + 2x)^{-1}$. At this stage, however, a common error was for candidates to integrate to give an expression in terms of natural logarithms. A significant number of candidates were unable to cope with substituting the rational limits to achieve the correct answer of $\frac{\pi}{12}$.

The vast majority of candidates were unable to gain any marks in part (b). Some candidates understood how the two diagrams were related to each other and were able to find the linear scale factor of 4. Few candidates then recognised that this scale factor needed to be cubed in order for them to go onto find the volume of the paperweight. Instead, a significant number of candidates applied the volume formula they used in part (a) with new limits of 0 and 3.

6. In part (a), most candidates realised that to find the shaded area they needed to integrate $3\sin(\frac{x}{2})$ with respect to x, and the majority of them produced an expression involving $\cos(\frac{x}{2})$; so gaining the first method mark. Surprisingly a significant number of candidates were unable to obtain the correct coefficient of -6, so thereby denying themselves of the final two accuracy marks. Most candidates were able to use limits correctly, though some assumed that cos0 is zero.
In part (b), whilst most candidates knew the correct formula for the volume required, there were numerous errors in subsequent work, revealing insufficient care in the use or understanding of trigonometry. The most common wrong starting point was for candidates to write y^2as $3\sin^2\left(\frac{x^2}{4}\right)$, $9\sin^2(\frac{x^2}{4})$or $3\sin^2(\frac{x}{2})$. Although some candidates thought that they could integrate $\sin^2(\frac{x}{2})$directly to give them an incorrect expression involving $\sin^3(\frac{x}{2})$, many realised that they needed to consider the identity $\cos 2A \equiv 1 - 2\sin^2 A$ and so gained a method mark. At this stage, a significant number of candidates found difficultly with rearranging this identity and using the substitution $A= \frac{x}{2}$ to give the identity $\sin^2\left(\frac{x}{2}\right) \equiv \frac{1-\cos x}{2}$. Almost all of those candidates who were able to substitute this identity into their volume expression proceeded to correct integration and a full and correct solution.
There were, however, a significant minority of candidates who used the method of integration by parts in part (b), but these candidates were usually not very successful in their attempts.

7. There were many excellent solutions to this question but also too many who did not know the formula for finding the volume of the solid. Candidates who successfully evaluated $\int_1^3 x^2 e^{2x}\, dx$ were able to gain 6 of the 8 marks, even if the formula used was $k\int y^2 \mathrm{d}x$ with $k \neq \pi$, but there were many candidates who made errors in the integration, ranging from the slips like sign errors and numerical errors to integrating by parts "in the wrong direction". An error with serious consequences for most who made it was to write $(x\,e^x)^2$ as $x^2\ e^{x^2}$; for some it was merely a notational problem and something could be salvaged but for most it presented a tricky problem !

8. For many candidates this proved to be the most testing question on the paper.

(a) This was attempted with some degree of success by most candidates and many scored full marks. Most errors occurred in squaring and simplifying *y*. Some of the integration was not good – candidates attempting to integrate a fraction by integrating numerator and denominator separately, a product by integrating separately and then finding the product, or a squared function by squaring the integral of the original function. In some cases candidates failed to recognise that 2/x integrated to a *log* function.

(b) Very few candidates knew the formula for the volume of a cone, so a further volume of revolution was often found. Candidates working with the incorrect equation for the line expended considerable time calculating an incorrect value for the volume. Many candidates produced answers that were clearly not dimensionally correct in (b), and hence lost all 3 marks. It was common to see expressions of the form "final volume = volume – area" or "final volume = volume – area^2"

Almost all of those candidates who were able to substitute this identity into their volume expression proceeded to correct integration and a full and correct solution.
There were, however, a significant minority of candidates who used the method of integration by parts in part (b), but these candidates were usually not very successful in their attempts.

9. Although there were a minority who thought that the volume required was $2\pi\int y^2\,\mathrm{d}x$, this is a question for which most candidates knew the correct procedure and it is disappointing to record that less that half were able to give completely correct solutions. Many were unable to square $\left(4x - \frac{6}{x}\right)$ correctly and the integral of $\frac{36}{x^2}$ gave difficulty, both $36\ln x^2$ and $\frac{36}{\frac{1}{3}x^3}$ being seen. Calculator errors were also frequently noted when simplifying the final fractions. These arose mainly from the incorrect use of brackets. A few candidates gave the final answer as an approximate decimal, failing to note that the question asked for an exact value of the volume.

10. In general, the attempts at part (a) were good and there was a large number of candidates who scored 6 or 7 marks. Even with poor squaring of $\left(\frac{x+2}{\sqrt{x}}\right)$ it was possible for candidates to gain 5 marks, which helped many, but some attempts at integration were not so kindly looked upon; a maximum of three marks was available for candidates who integrated the numerator and denominator separately or who produced $\left(\frac{x^3}{3} + 2x^2 + 4x\right)\ln x$. The majority of candidates gained the mark in part (b), but, surprisingly, correct reasoning in part (c) was uncommon. The most common approach was to approximate the support to a cylinder of radius 6cm and height 6cm, so $\pi(6)^2 \text{ x } 6 = 678.6 \approx 630$ was seen frequently

11. In part (a) errors in indices were seen and in part (b) many found expanding $\left(\frac{8}{x} - x^2\right)^2$ a stumbling block. In the integration, as expected, integrating $\frac{64}{x^2}$ was the major source of error. However, for the majority, the methods and formulae needed for this question were well known and there were many completely correct solutions.

12. The key to success in part (a) was to realize the need to differentiate the curve. Many weaker candidates did not appreciate this but there were many good solutions to this part. In part (b) many candidates did not realize that they needed to combine the result in part (a) with $y = 1 + \frac{c}{p}$ and circular arguments that started by assuming $c = 4$ and only used one of these statements were seen. Part (c) was answered very well and many fully correct solutions were seen. The volume formula was well known and working exactly caused few problems. There were a few errors in squaring, where the $\frac{8}{x}$ term was missing, and some thought that the integral of $\frac{16}{x^2}$ was $16 \ln x^2$.

13. Most candidates made some attempt to differentiate $x\sqrt{\sin x}$, with varying degrees of success. $\sqrt{\sin x} + x\sqrt{\cos x}$ was the most common wrong answer. Having struggled with the differentiation, several went no further with this part. It was surprising to see many candidates with a correct equation who were not able to tidy up the √ terms to reach the required result.

Most candidates went on to make an attempt at $\int \pi y^2 \, dx$. The integration by parts was generally well done, but there were many of the predictable sign errors, and several candidates were clearly not expecting to have to apply the method twice in order to reach the answer. A lot of quite good candidates did not get to the correct final answer, as there were a number of errors when substituting the limits.

14. It was pleasing to see most candidates applying the volume of revolution formula correctly. However, although the question was well attempted, with the majority of candidates scoring at least 5 marks despite errors listed below, fully correct solutions were usually only seen from the better candidates.

Algebraic errors arose in expanding $\left(1 + \frac{1}{2\sqrt{x}}\right)^2$; the most common wrong attempts being

$1 + \frac{1}{4x}$, $1 + 4x^{-\frac{1}{2}} + 4x^{-1}$, $1 + \frac{1}{4}x + \frac{1}{4}x$ and $1 + \frac{1}{2}x^{-1} + \frac{1}{4}x^{-\frac{1}{4}}$.

Common errors in integration were $\int \frac{1}{4x}\,\mathrm{d}x = \ln 4x$ and $\int \frac{1}{\sqrt{x}}\,\mathrm{d}x = \ln\sqrt{x}$.

15. This question involved differentiation using the product rule in part (a) and integration using parts in part (b). It was answered well, with most of the difficulties being caused by the use of indices and the associated algebra. Some candidates wasted time in part (a) by finding the y-coordinate which was not requested. A sizeable proportion of the candidates misquoted the formula for volume of revolution.

16. No Report available for this question.